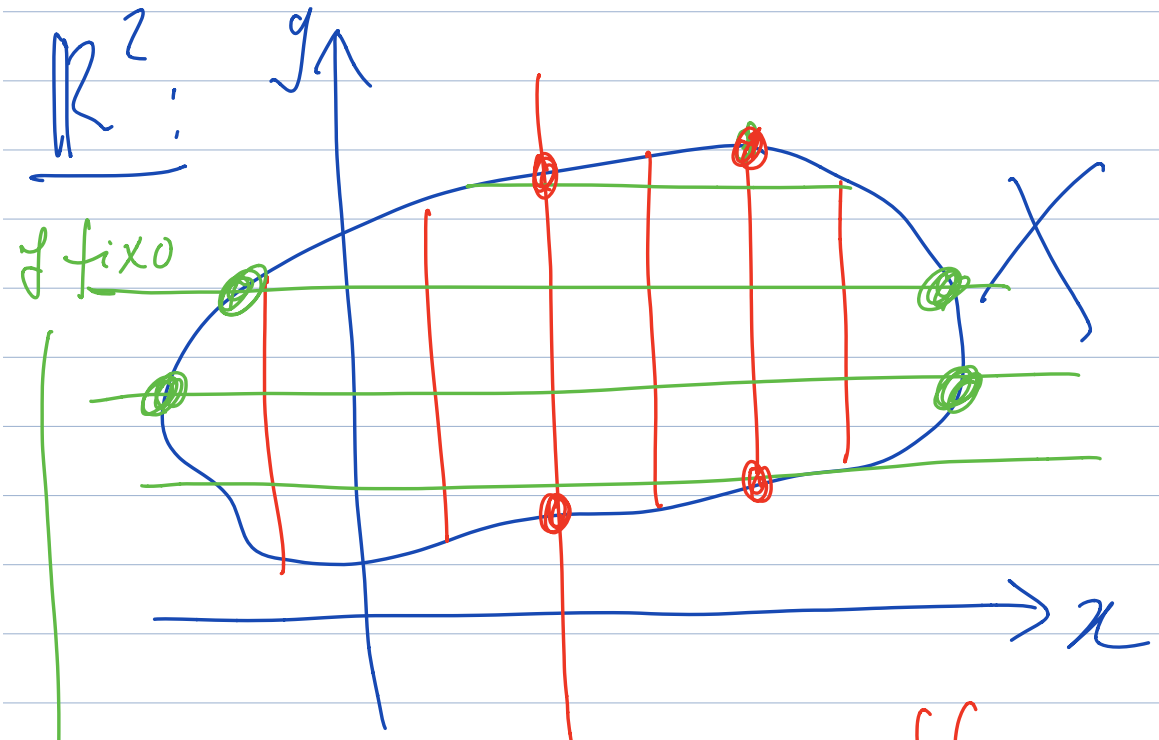
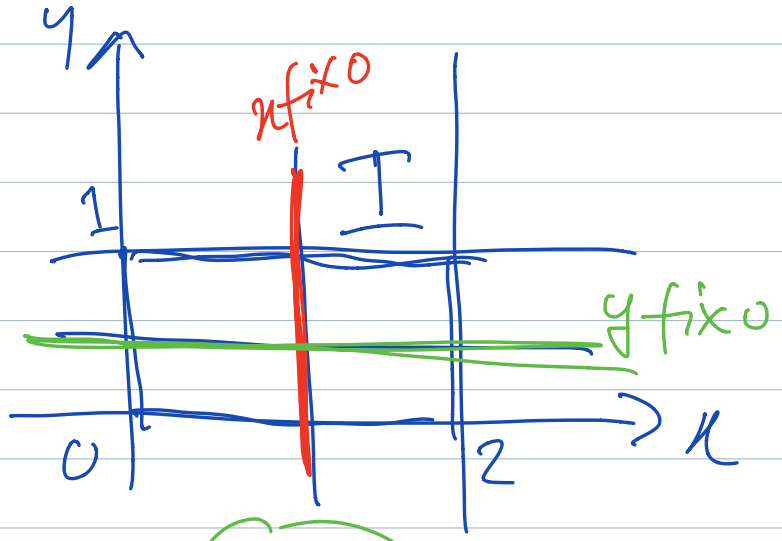
# CDI-II - Prática 14/4/21

## Ficha 6: Fubini

$\mathbb{R}^2$:



1-6)




$$\iint_I f(x,y)\, dx\,dy \quad \text{ou} \quad dy\,dx$$

$$\frac{d}{dy}\operatorname{sen}(xy) = x\cos(xy)$$

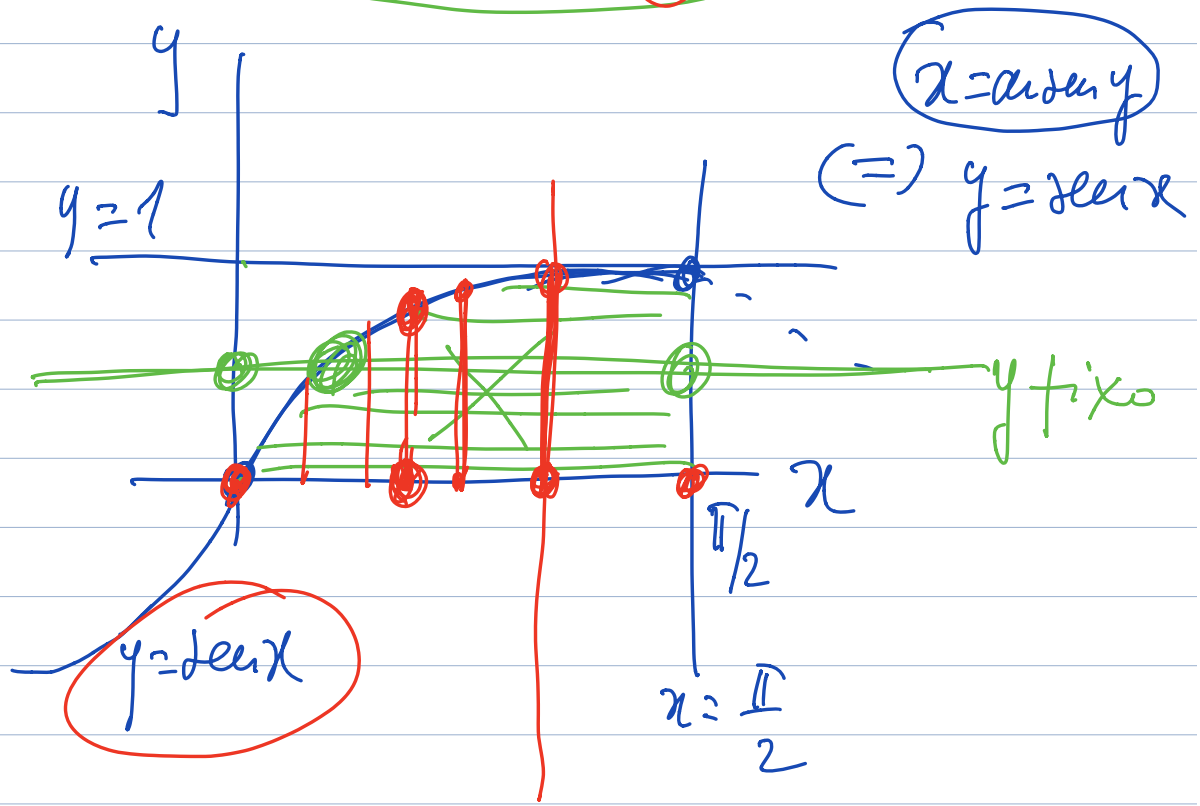
$$\int_0^2 \left( \int_0^1 x\cos(xy)\,dy \right) dx = \int_0^2 \left( \operatorname{sen}(xy)\Big|_{y=0}^{1} \right) dx$$

$$= \int_0^2 \operatorname{sen} x\, dx = 1 - \cos(2) \quad //$$

2-b) $0 \le y \le 1$

$\arcsin y \le x \le \frac{\pi}{2}$

$\times$ $dx\,dy$




$$\int_0^{\frac{\pi}{2}} \left( \int_0^{\operatorname{sen} x} y \operatorname{sen} x \, dy \right) dx =$$

$$= \int_0^{\frac{\pi}{2}} \left( \frac{\operatorname{sen} x}{2} \, y^2 \Big|_0^{\operatorname{sen} x} \right) dx =$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \operatorname{sen} x \cdot \operatorname{sen}^2 x \, dx$$

$$= \frac{1}{2} \int_0^{\pi/2} \operatorname{sen} x \left(1 - \cos^2 x\right) dx$$

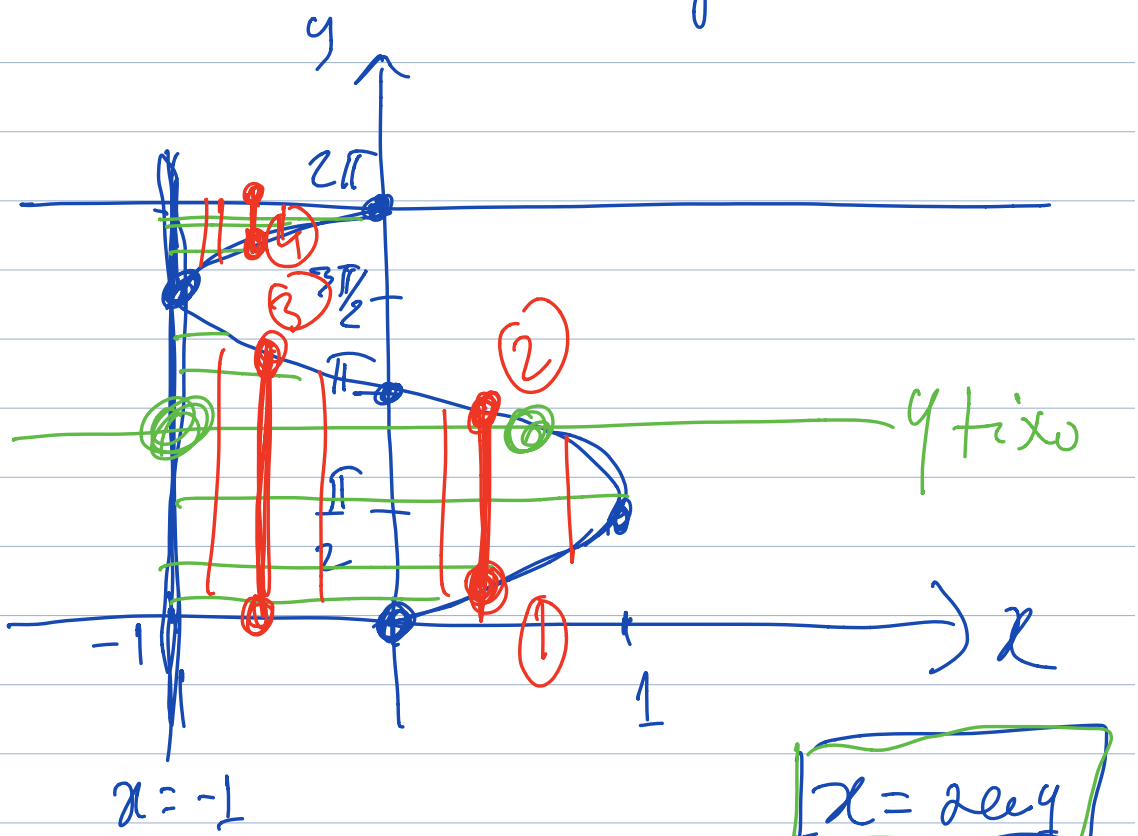
$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \operatorname{sen} x \, dx - \frac{1}{6} \int_0^{\frac{\pi}{2}} 3 \operatorname{sen} x \cos^2 x \, dx$$

$\left(\cos^3 x\right) = 3\cos^2 x \, (-\operatorname{sen} x)$

$$= -\frac{1}{2} + \frac{1}{6} \cos^3 x \Big|_{x=0}^{\frac{\pi}{2}}$$

$$= -\frac{1}{2} - \frac{1}{6} \quad //$$

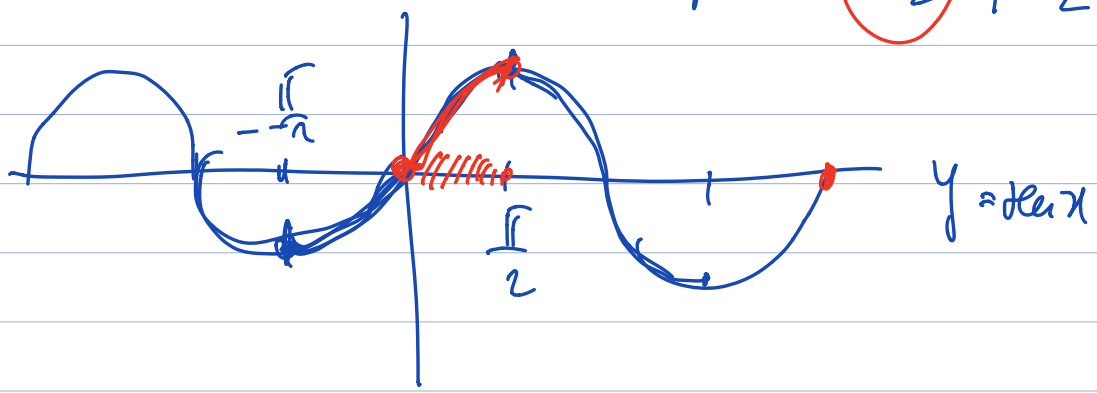
3-c) $\begin{cases} 0 < y < 2\pi \\ -1 < x < 2\operatorname{sen} y \end{cases}$ X

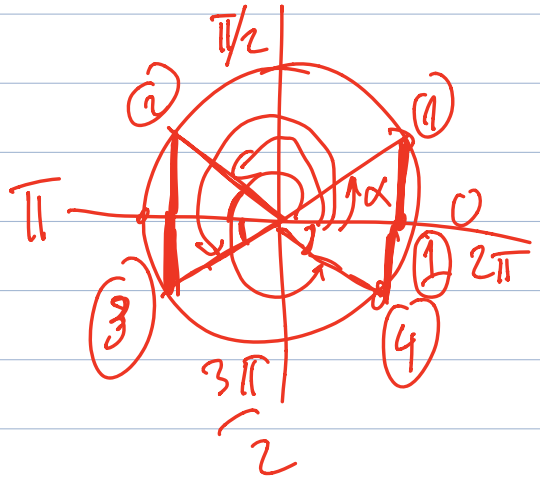



$x = 2\operatorname{sen} y$

$\Leftrightarrow$ $y = \operatorname{arsen} x$

para $-\frac{\pi}{2} < y < \frac{\pi}{2}$

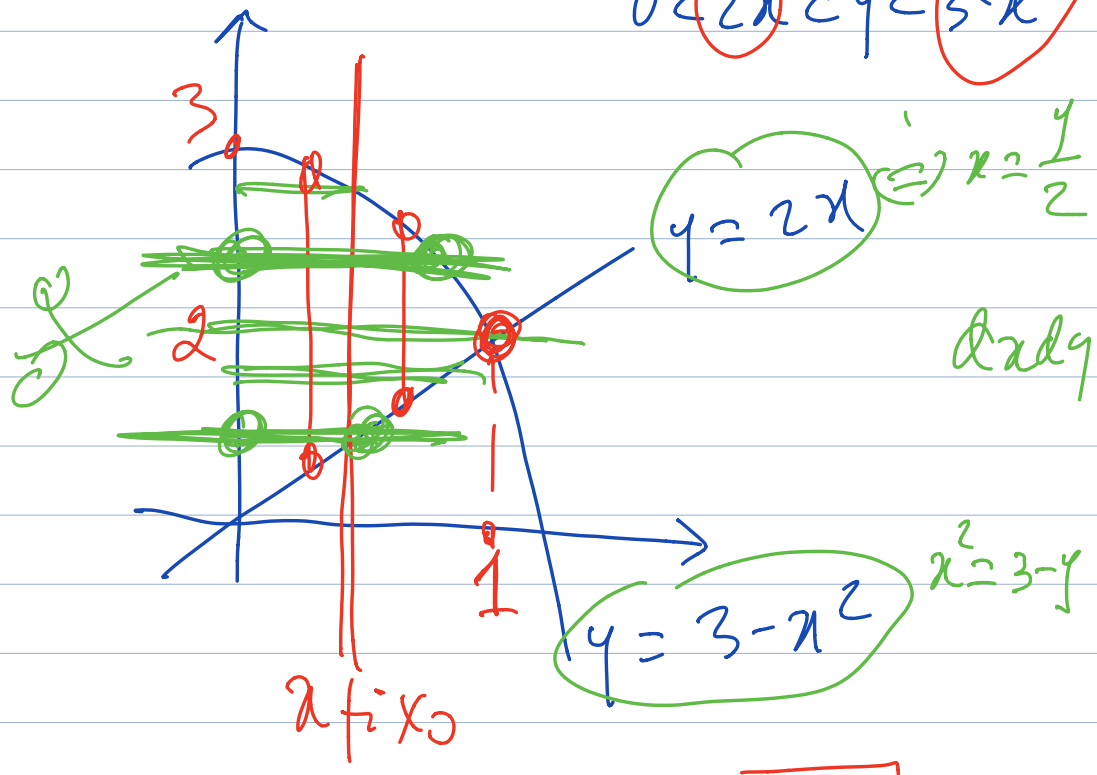





II

$0 < 2x < y < 3 - x^2$

y-




dydx

$0 < \sqrt{2x} < y < \sqrt{3 - x^2}$

$0 < 2x < 3 - x^2$

$$2x < 3 - x^2$$

$$x^2 + 2x - 3 < 0$$

$$x^2 + 2x - 3 = 0$$

$$x = \frac{-2 \pm \sqrt{4+12}}{2}$$

1

$-3$

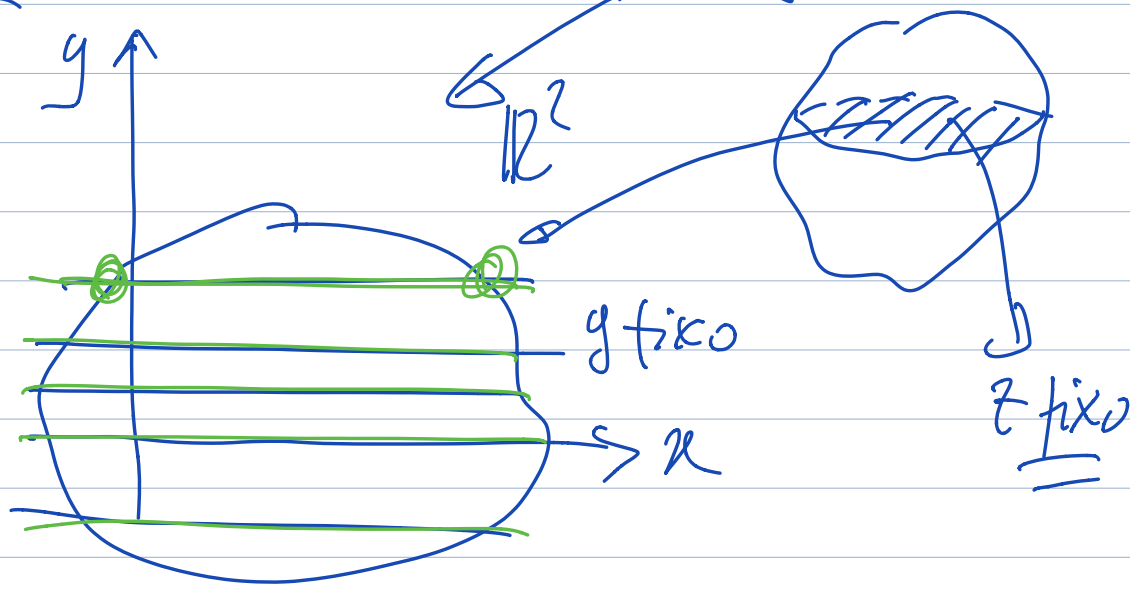
$$\int_0^2 \left( \int_0^{\frac{y}{2}} dx \right) dy + \int_2^3 \left( \int_0^{\sqrt{3-y}} dx \right) dy$$

etc.

$$\int_0^1 \left( \int_{2x}^{3-x^2} dy \right) dx = \int_0^1 (3 - x^2 - 2x)\, dy$$

$$= 3 - \frac{1}{3} - 1 \quad //$$

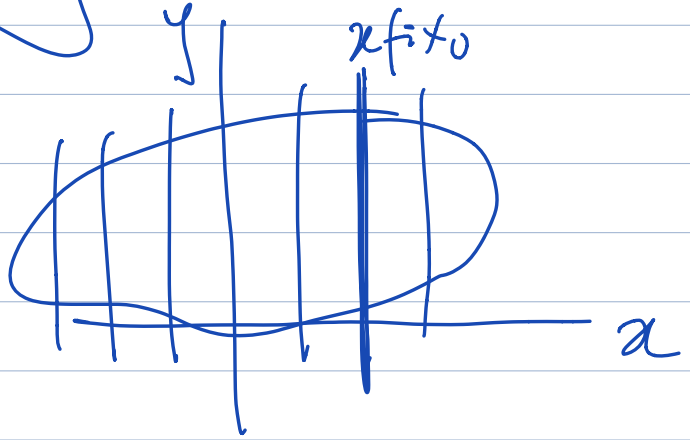
$\mathbb{R}^3$: $dx\,dy\,dz$ $\quad$ $\mathbb{R}^3$ | $z$ fixo $\rightarrow y(t) \rightarrow x(y,t)$




$\Downarrow$

5-a) $dy\,dx\,dz$

z fixo $\rightarrow x(z) \rightarrow y(x,z)$

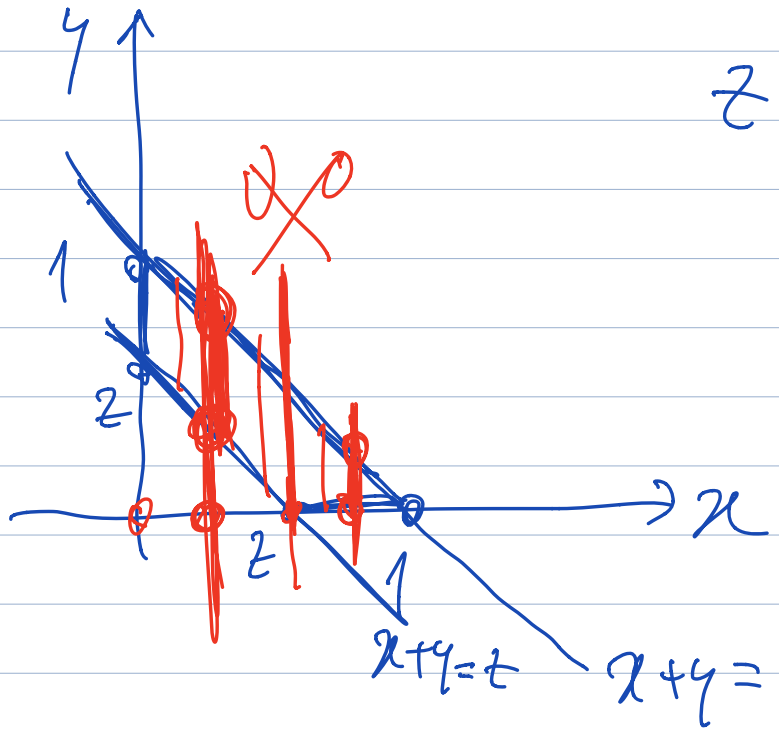


$x, y > 0$ , $x + y < 1$ : X

$0 < z < x + y$

z fixo: $0 < z < x + y < 1$

$0 < z < 1$ fixo




$z < x + y < 1$

$x + y = 1$ ✓

$x + y = z$

$dy\,dx\,dz$

$$\int_0^1 \left( \int_0^z \left( \int_{z-x}^{1-x} dy \right) dx \right) dz + \int_0^1 \left( \int_z^1 \left( \int_0^{1-x} dy \right) dx \right) dz$$

7-a) $dy\,dx\,dz$

$x+y+2z<1$ $\qquad x,y>0$

$x+y-2z<1$

z fixo

$2z < x+y+2z < 1$

$z < \frac{1}{2}$

$-2z < x+y-2z < 1$

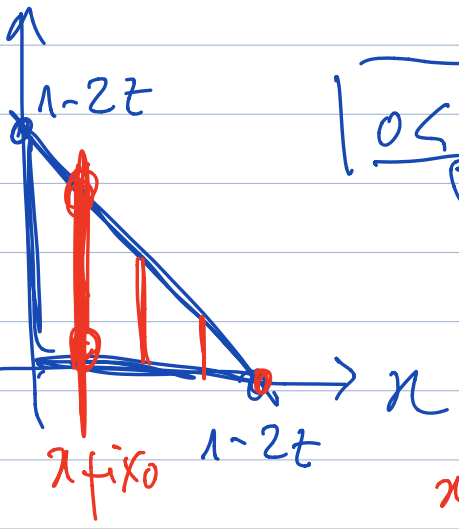
$z > -\frac{1}{2}$

$-\frac{1}{2} < z < \frac{1}{2}$

$0<z<\frac{1}{2}$

$x+y<1-2z$

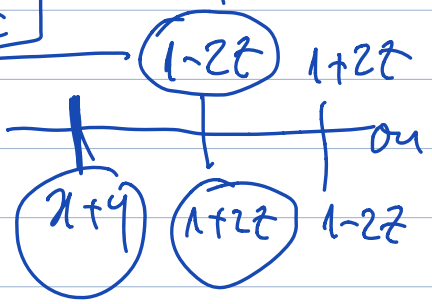
$x>0, y>0$




$0<4z$

$x+y<1-2z$

$x+y<1+2z$

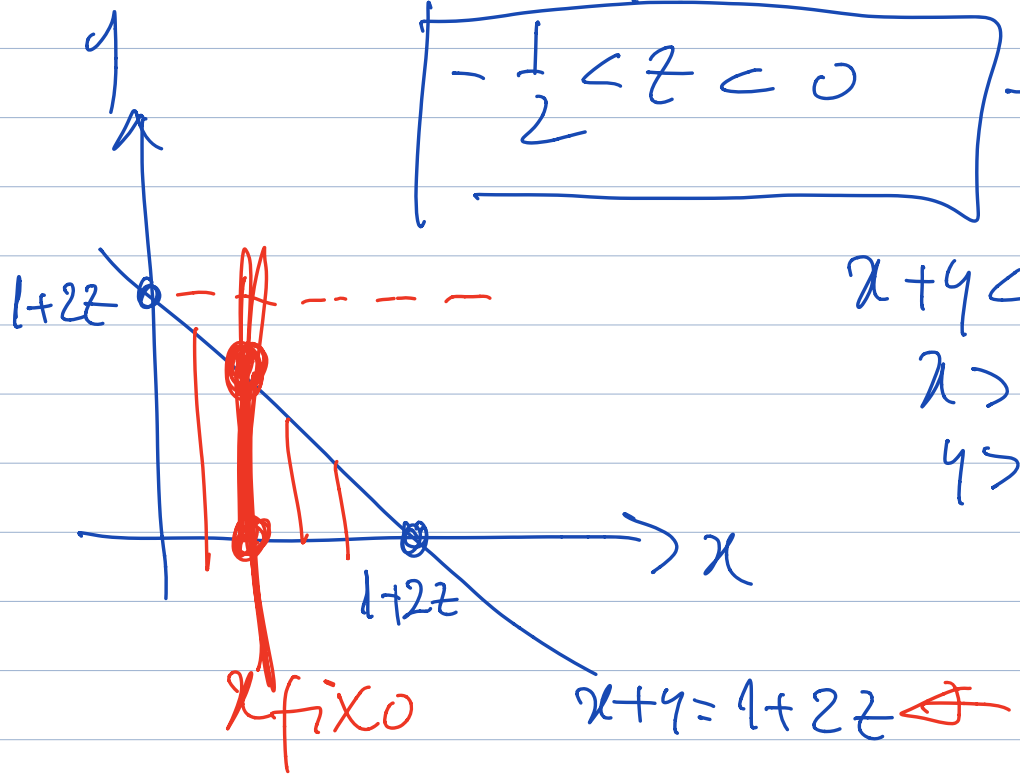



$x+y<1-2z$

2º caso $1+2z < 1-2z$

$4z < 0$

$\boxed{-\frac{1}{2} < z < 0}$ fixo




$x+y < 1+2z$

$x > 0$

$y > 0$

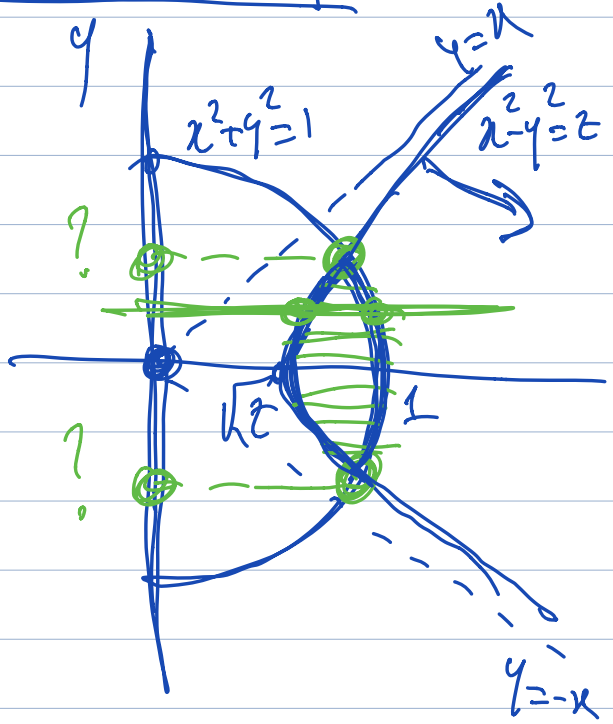
$$\int_{-\frac{1}{2}}^{0} \left( \int_{0}^{1+2z} \left( \int_{0}^{1+2z-x} dy \right) dx \right) dz +$$

$$+ \int_{0}^{\frac{1}{2}} \left( \int_{0}^{1-2z} \left( \int_{0}^{1-2z-x} dy \right) dx \right) dz$$

6-b) $\begin{cases} x^2+y^2<1 \\ 0<z<x^2-y^2, \quad x>0 \end{cases}$ : X

$z$ fixo: $0<z<x^2-y^2<x^2<x^2+y^2<1$

$\boxed{0<z<1}$ fixo




$$\begin{cases} x>0 \\ x^2+y^2<1 \\ x^2-y^2>z \end{cases}$$

$\boxed{x^2-y^2=z}$

$>0$ $>0$

$(x+y)(x-y)=z$